ECE 490: Introduction to Optimization **Fall 2018**

Homework 4

Instructor: Bin Hu Due date: November 8, 2018

1. Given $x \in \mathbb{R}^p$, we denote its i-th entry as x^i. Suppose $X = \{x : a^i \leq x^i \leq b^i, \forall i \in \{1, 2, \ldots, p\}\}$. For an arbitrary $y \in \mathbb{R}^p$, what is the projection of y onto X? Denote the projection of y onto X as $[y]_+$. Write out each entry of $[y]_+$ as a function of y^i, a^i, and b^i.

2. Programming Assignment

Implement ISTA for solving the following LASSO problem

$$\min_{x \in \mathbb{R}^p} \frac{1}{2}\|Ax - b\|^2 + \mu\|x\|_1$$

where $A \in \mathbb{R}^{m \times p}$, $x \in \mathbb{R}^p$, and $b \in \mathbb{R}^m$. $\mu \geq 0$ is some positive parameter; m should be significantly smaller than p (which means the number of data points is much less than the number of features and this is typical for biological applications). The data matrix A and the vector b can be randomly generated.

(a) Please explicitly write down the algorithm steps for ISTA.

(b) For a given problem instance (i.e., fixed A and b), please consider using different values of μ and discuss the results (i.e., sparsity (how many entries of your solution are zero), error $\frac{1}{2}\|Ax^* - b\|^2$, etc). Also discuss how the stepsize selection affects your results. Vary p and m in your simulations. Use $p = 500$ and $p = 1500$. Try $m \approx \frac{1}{3}p$ and $m \approx \frac{1}{4}p$. Are the results consistent? Notice for the above problem we may not have strong convexity and the convergence can be sublinear.