ECE 490: Introduction to Optimization	**Fall 2018**

Homework 5

Instructor: Bin Hu | Due date: November 15, 2018

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(a) Consider the constrained minimization problem:

$$\begin{aligned} \text{minimize} \quad & x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 \\ \text{subject to} \quad & x_1 + x_2 + x_3 + x_4 + x_5 = 5. \end{aligned}$$

Determine all the local mins for the above problem.

(b) Consider minimizing the function $f(x) = \frac{1}{2}\|x\|^2 + \frac{1}{2}(a^\mathsf{T} x - b)^2$ where $a \in \mathbb{R}^p$, $b \in \mathbb{R}$, and $x \in \mathbb{R}^p$. This problem can be rewritten as

$$\begin{aligned} \text{minimize} \quad & \tfrac{1}{2}\|x\|^2 + \tfrac{1}{2}(y - b)^2 \\ \text{subject to} \quad & a^\mathsf{T} x = y \end{aligned}$$

Write out the Lagrangian $L(x, y, \lambda)$ for the above problem and calculate the related dual function $D(\lambda) = \min_{x,y} L(x, y, \lambda)$.

2. (a) Consider the basis pursuit problem

$$\begin{aligned} \text{minimize} \quad & \|x\|_1 \\ \text{subject to} \quad & Ax = b \end{aligned}$$

This problem can be rewritten as the following form:

$$\begin{aligned} \text{minimize} \quad & f(x) + g(y) \\ \text{subject to} \quad & x - y = 0 \end{aligned} \tag{1}$$

where $f(x)$ is an indicator function of the set $\{x : Ax = b\}$, and $g(y) = \|y\|_1$. Recall the indicator function satisfies $f(x) = 0$ if $Ax = b$ and $f(x) = \infty$ if $Ax \neq b$. The augmented Lagrangian function is given as

$$L_\rho(x, y, \lambda) = f(x) + g(y) + \lambda^\mathsf{T}(x - y) + \frac{\rho}{2}\|x - y\|^2 \tag{2}$$

Your task is to write out the ADMM update formula for (1) using the projection operator onto the set $\{x : Ax = b\}$ and the shrinkage operator. Specifically, write out x_{k+1}, y_{k+1}, and λ_{k+1} as functions of x_k, y_k, and λ_k. Simplify the arg min operation using the projection operator and the shrinkage operator.

(b) Consider the following least square problem $\min_x \sum_{i=1}^n \frac{1}{2}(a_i^\mathsf{T} x - b_i)^2$ where $a_i \in \mathbb{R}^p$, $b_i \in \mathbb{R}$, and $x \in \mathbb{R}^p$. This problem can be rewritten as

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^n f_i(x^i) \\ \text{subject to} & x^i - y = 0, \forall i \in \{1, 2, \ldots, n\} \end{array}$$

where $f_i(x^i) = \frac{1}{2}(a_i^\mathsf{T} x^i - b_i)^2$, and $x^i \in \mathbb{R}^p$ is a vector having the same dimension as a_i. The augmented Lagrangian is given by

$$L_\rho = \sum_{i=1}^n \left\{ f_i(x^i) + (\lambda^i)^\mathsf{T}(x^i - y) + \frac{\rho}{2}\|x^i - y\|^2 \right\}$$

Your task is to write out the ADMM update formula for the above problem. Specifically, express x_{k+1}^i, y_{k+1}, and λ_{k+1}^i as functions of x_k^i, y_k, λ_k^i, a_i, b_i and ρ.