ECE 490: Introduction to Optimization

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Due date: December 4, 2018

1. Consider the constrained minimization problem:

$$
\begin{aligned}
\operatorname{minimize} & (x-a)^{2}+(y-b)^{2}+x y \\
\text { subject to } & 0 \leq x \leq 1,0 \leq y \leq 1
\end{aligned}
$$

Here $a$ and $b$ are some given scalers. Determine all the local mins for the above problem.
2. Consider the optimization problem

$$
\begin{aligned}
\operatorname{minimize} & 2 x^{2}+1 \\
\text { subject to } & (x-1)(x-5) \leq 0
\end{aligned}
$$

where $x$ is a scalar.
(a) What is the optimal solution for the above problem?
(b) What is the dual problem? Solve the dual problem. Compare the solutions of the primal and dual problems. Does the strong duality hold?
3. Consider the minimization problem

$$
\begin{aligned}
\operatorname{minimize} & x^{\top} Q x \\
\text { subject to } & A x \leq b
\end{aligned}
$$

where $Q \in \mathbb{R}^{n \times n}$ is positive definite. What is the dual problem for the above problem?
4. Consider the following SDP

$$
\begin{aligned}
\text { minimize } & 0 \\
\text { subject to } & {\left[\begin{array}{cc}
A^{\top} P+P A & P B \\
B^{\top} P & 0
\end{array}\right]-X \leq 0 }
\end{aligned}
$$

where $A, B$, and $X$ are given. $P$ is the decision variable. What is the dual problem for the above SDP?

