ECE 490: Introduction to Optimization **Fall 2018**

Homework 7

Instructor: Bin Hu Due date: December 11, 2018

1. Consider the constrained minimization problem:

$$\begin{aligned} \text{minimize} \quad & x^2 + y^2 \\ \text{subject to} \quad & x - 1 \geq 0 \\ & y + 1 \geq 0 \end{aligned}$$

What is the optimal solution for this problem?

Now apply the barrier function method that iterates as

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \arg\min_{x,y} \left\{ x^2 + y^2 - \varepsilon_k \ln(x-1) - \varepsilon_k \ln(y+1) \right\}$$

where ε_k decreases to 0 as k increases. Does the above barrier function converge? Prove your conclusion.

2. Consider the constrained minimization problem:

$$\begin{aligned} \text{minimize} \quad & x - 2y \\ \text{subject to} \quad & 1 + x - y^2 \geq 0 \\ & y \geq 0 \end{aligned}$$

What is the optimal solution for this problem?

Now apply the barrier function method that iterates as

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \arg\min_{x,y} \left\{ x - 2y - \varepsilon_k \ln(1 + x - y^2) - \varepsilon_k \ln(y) \right\}$$

where ε_k decreases to 0 as k increases. Does the above barrier function converge? Prove your conclusion.

3. Programming Assignment

Consider the following linear programming problem

$$\begin{aligned} \text{minimize} \quad & c^{\mathsf{T}} x \\ \text{subject to} \quad & Ax \geq b \end{aligned}$$

where $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, and $\text{rank}(A) = n$. Use $m = 100$ and $n = 50$. Generate A, b, and c randomly. Make sure the rank of A is n. Implement the short-step interior point method covered in the class. Specifically, for every k, just perform one-step Newton update for f_{ε_k} and then shrinks ε_k by a constant factor. Plot the results and discuss how ε_k affects the convergence. To make sure the problem is strictly feasible, you can just uniformly sample the entries of b from [-2, -0.5]. Then $x = 0$ is a strictly feasible point for the problem.