ECE 490 (Introduction to Optimization) – In-Class Problem Discussions

03/01/2022

Problem 1. Consider the following quadratic function:

$$f(x) = f(x_1, x_2) = 2x_1^2 + 2x_1x_2 + 2x_2^2 - 5x_2 + 2$$

(a) If you use the steepest descent method to minimize the above function, how to choose the stepsize? Provide some reasons. What type of convergence behaviors will you get?

(b) If Newton's method is applied, what type of convergence behavior will you get?

(c) Find the minimum and maximum of f over $\mathbb{R}^2$ if they exist.

(d) In general, what are the cons/pros of the steepest descent method when compared with Newton's method?

Problem 2. True or False. Provide reasons.

(a) If S_1 and S_2 are two convex sets, then $S_1 \cup S_2$ is convex.

(b) If S_1 and S_2 are two convex sets, then $S_1 \cap S_2$ is convex.

(c) If f and g are both convex, then $f(g(x))$ is also convex.

(d) Gradient descent algorithm always converges to a local optimizer for a smooth function.

(e) For a strictly convex function, Newton's method always converges to a minimizer, starting from any point within the domain of this function.

Problem 3. Convergence under PL condition: Suppose f is L-smooth and also satisfies the PL condition:

$$f(x) - f(x^*) \leq \frac{1}{2\mu}||\nabla f(x)||^2,$$

where x^* is the unique global min of f. If we apply the steepest descent method to minimize f, does $f(x_k)$ converge to $f(x^*)$? Try to prove a linear convergence bound in the following form:

$$f(x_k) - f(x^*) \leq \rho^k C$$

where $0 < \rho < 1$ and C are fixed constants. What type of stepsize shall we use?