ECE 490 (Introduction to Optimization) – Homework 5

Problem 1. (20 points) Using the KKT conditions, find the global minimum for the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & 2x_1^2 + 2x_1x_2 + x_2^2 - 10x_1 - 10x_2 \\ \text{subject to} \quad & x_1^2 + x_2^2 \leq 5, \\ & 3x_1 + x_2 \leq 6 \end{aligned} \tag{1}$$

Problem 2. (20 points) Using the KKT conditions, find the global minimum for the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & x_1^2 + x_2^2 - 6x_1 - 14x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 2, \\ & 2x_1 + x_2 \leq 3 \end{aligned} \tag{2}$$

Problem 3. (20 points) Using the KKT conditions, find the global minimum for the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & -\log(1 + x_n) - \sum_{i=1}^{n-1} \log(x_i) \\ \text{subject to} \quad & x_i + x_n \leq 1, \;\; i = 1, \cdots, n-1 \\ & x_i \geq 0, \;\; i = 1, \cdots, n \end{aligned} \tag{3}$$

Hint: First note that x_i cannot be 0 for $i = 1, \cdots, n-1$, i.e., the constraints $x_i \geq 0$ are inactive for $i = 1, \cdots, n-1$. Then show that the constraints $x_i + x_n \leq 1, \; i = 1, \cdots, n-1$ all have to be active. Then you will be left with only two cases to consider, i.e., whether the constraint $x_n \geq 0$ is active or inactive.

Problem 4. (20 points) Consider the constrained minimization problem:

$$\begin{aligned} \text{minimize} \quad & x^2 + y^2 \\ \text{subject to} \quad & x - 2 \geq 0 \\ & y + 1 \geq 0 \end{aligned}$$

What is the optimal solution for this problem?

Now apply the barrier method that iterates as

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \arg\min_{x,y} \left\{ x^2 + y^2 - \epsilon_k \ln(x-2) - \epsilon_k \ln(y+1) \right\}$$

where ϵ_k decreases to 0 as k increases. Does the above barrier method converge to the optimal solution? Prove your conclusion.

Problem 5. (20 points) Consider the constrained minimization problem:

$$\begin{aligned} \text{minimize} \quad & x^2 + y^2 \\ \text{subject to} \quad & x + y = 4 \end{aligned}$$

What is the optimal solution for this problem?

Now apply the basic quadratic penalty method from Lecture Note 18 to the optimization problem. Does the penalty method converge to the optimal solution? Prove your conclusion.