ECE 490 (Introduction to Optimization) – Homework 6

Problem 1. Consider the function $g(y, z)$, with $y \in \mathcal{Y}$ and $z \in \mathcal{Z}$. Assuming both sides of the inequality exist, show that:

$$\max_{y\in\mathcal{Y}} \min_{z\in\mathcal{Z}} g(y,z) \leq \min_{z\in\mathcal{Z}} \max_{y\in\mathcal{Y}} g(y,z).$$

Problem 2. Find the dual of the following linear program:

$$\begin{aligned} \text{minimize} \quad & x_1 + x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \geq 1, \\ & 3x_1 + x_2 \leq 5, \\ & -x_1 + x_2 \leq 8. \end{aligned} \tag{1}$$

Does the strong duality hold?

Problem 3. Consider the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & x^\top Q x \\ \text{subject to} \quad & Ax = b, \end{aligned} \tag{2}$$

where $Q \in \mathbb{R}^{n\times n}$ is positive definite. What is the dual problem for (2)

Problem 4. Consider the optimization problem

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & h(x) = 0 \end{aligned} \tag{3}$$

Recall that the augmented Lagrangian is defined as

$$L_c(x,\lambda) = f(x) + \lambda^\top h(x) + c\|h(x)\|^2, \;\; \lambda \in \mathbb{R}^m, \;\; c > 0.$$

Now suppose $\{c_k\}$ is a sequence of positive numbers that increases to ∞ as $k \to \infty$, and let

$$x^{(k)} \in \arg\min_x L_{c_k}(x,\lambda)$$

Then show that every limit point $\bar{x}$ of the sequence $\{x^{(k)}\}$ is a global minimum for (3) (assuming that the global min exists).

Problem 5. Consider the constrained minimization problem

$$\begin{aligned} \text{minimize} \quad & x + y \\ \text{subject to} \quad & z \geq 0, \\ & x + z = 10, \\ & z^2 \leq x + y \end{aligned}$$

Is this a convex optimization problem? Explain your answer. (You don't have to solve the minimization problem.)