ECE 490 (Introduction to Optimization) – Homework 7

Problem 1. Prove the following properties of subgradients (here f, f_1 and f_2 are convex functions):

(a) Scaling: For scalar $a > 0$, $\partial(af) = a\partial f$, i.e., g is a subgradient of f at x if and only if ag is a subgradient of af at x.

(b) Addition: If g_1 is a subgradient of f_1 at x, and g_2 is a subgradient of f_2 at x, then $g_1 + g_2$ is subgradient of $f_1 + f_2$ at x.

(c) Affine Combination: Let $h(x) = f(Ax+b)$, with A being a square, invertible matrix. Then $\partial h(x) = A^\top \partial f(Ax+b)$, i.e., g is a subgradient of f at Ax + b if and only if $A^\top g$ is a subgradient of h at x.

Problem 2. Consider the following convex function:

$$f(x) = f(x_1, x_2, x_3) = |x_1| + |x_2| + |x_3|$$

Write down your conjecture for the subdifferential $\partial f(x)$ for $(x_1, x_2, x_3) = (0, 0, 0)$. Prove that your conjecture is indeed correct. (Don't forget to give the converse argument.)

Problem 3. If we apply the subgradient method with stepsize $\alpha_k = \frac{1}{\sqrt{k+1}}$ to minimize the function in Problem 2, does the method always converge to the global minimum?