ECE 490 (Introduction to Optimization) – A Practice Problem

Problem 1. Define $f : \mathbb{R}^2 \to \mathbb{R}$ as

$$f(x_1, x_2) = x_1^2 + 2\frac{1-\epsilon}{1+\epsilon}x_1x_2 + x_2^2$$

with $0 < \epsilon < 1$. Now, we consider the minimization problem of f, i.e. $\min_{x_1,x_2} f(x_1, x_2)$.

(a) What are the minimizers of f?

(b) Find the largest $m > 0$ and the smallest $M > 0$ in terms of ϵ such that

$$mI \preceq \nabla^2 f(x_1, x_2) \preceq MI$$

for all (x_1, x_2), where I is the identity matrix. Find the condition number of $\nabla^2 f$ given by $\kappa := \frac{M}{m}$ in terms of ϵ.

(c) How does κ change as ϵ decreases to 0. Do you expect gradient descent to converge faster or slower as ϵ decreases to 0?